Študentská vedecká konferencia FMFI UK, Bratislava, 2023, pp. 300–301
ISBN 978-80-8147-136-0, © 2023 Manaswita Das, Martin Čavarga, Andrej Ferko

On Planar Hamiltonian Triangulation (Not) Containing the Minimum Hamiltonian Cycle (TSC); Utilizing Deterministic, Stochastic, and Natural Approaches

Manaswita Das[1]* Martin Čavarga[1†] Andrej Ferko[1‡]
[1]KAG FMFI UK, Mlynská dolina, 842 48 Bratislava

1 Introduction

In 1978, Michael B. Dillencourt proved that Traveling Salesman Cycles are not always Subgraphs of Delaunay Triangulations or, of Minimum Weight Triangulations [Dillencourt, 1987]. TSC $\not\subset$ DT. TSC $\not\subset$ MWT. It implies a research question 'Which triangulation for given points contains the TSC solution?'

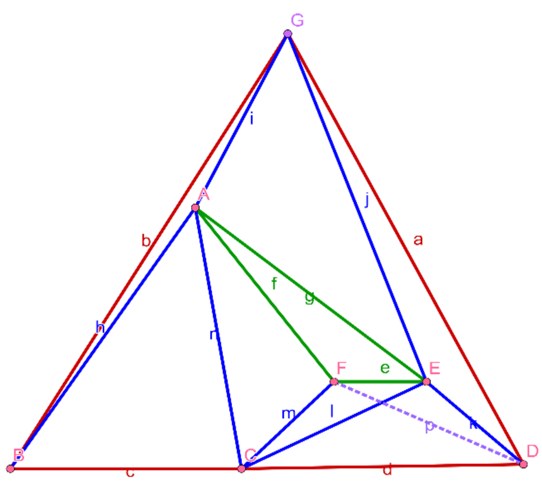

Figure 1: Dillencourt-7 dataset S = {A (4, 6), B (0, 0), C (5, 0), D (11.1, 0.1), E (9, 2), F (7, 2), G (6, 10)}. The 14 edges form both DT and MWT. The colors vividly represent the sequential stages of Dash Triangulation.

2 Constructing the Dillencourt-7 Triangulation

The extremal edges of the convex hull, which are highlighted in red in Figure 1, cannot be flipped or replaced with the opposite diagonal of the convex quadrilateral. These edges are referred to as *Unavoidable* edges. For the given set of points, denoted as S, the unavoidable edges are AB, AG, AF, FE, ED, and GE. Among these unavoidable edges, the edge with the shortest length is FE, which is known as the Closest Pair. In the Greedy Triangulation algorithm, the initial step involves inserting the edge FE. Remarkably, in this particular case, we observe that the Delaunay Triangulation (DT), the Minimum Weight Triangulation (MWT), and the Greedy Triangulation (GT) produce identical results. In our current research, we have embarked on the exploration of the triangulation of S, utilizing three distinct approaches: deterministic, stochastic, and natural processes.

Among these methods, the deterministic procedure we have developed is named Dash. The initial step of Dash is the construction of *Nested Convex Hulls*, with a depth of 2, giving rise to an outer hull composed of points B, C, D, and G, and an inner hull consisting of points A, E, and F (both hulls are indicated by edges with Red and Green respectively in Fig.1). This nesting approach allows us to capture the underlying complexity of the point set in a comprehensive manner. With the nested hulls in place, the second step proceeds with triangulating the annular region that emerges between them. This is achieved by applying the Delaunay Triangulation (edges are indicated with the Blue edges in Fig. 1). Here comes the third and final step. Within the triangulated annulus, an edge is selected to flip (indicated with the 'dashed' purple edge), keeping in mind the minimization of the total length of the path, resulting in the inclusion of the Traveling Salesman Cycle (TSC), BCFDEGAB.

The presence of the Hamiltonian path in the dual of triangulation allows for mesh stripification, i.e. the triangles can be organized in a stripe for real-time rendering. Moreover, both triangulations—those with and without the TSC—can be considered OPT (Onion Peeling) triangulations. This signifies that they effectively peel away the layers of complexity in the point set, revealing its essential structure.

The stripe is formed by the list of pairwise adjacent triangles FEA, AGE, DGE, CDE, CEF, ACF, ABC, and ABG.

* manaswita.das@fmph.uniba.sk
† martin.cavarga@fmph.uniba.sk
‡ ferko@fmph.uniba.sk

To enable a clear distinction between the subgraphs, we have proposed a novel matrix and color encoding scheme. This encoding method serves as a valuable tool to differentiate and categorize the two subgraphs, offering new avenues for analysis and exploration of the triangulation data.

Through the utilization of Dash, our deterministic procedure, we are able to construct triangulations that not only incorporate the TSC but also allow for mesh stripification. Furthermore, these triangulations exhibit the desirable qualities of being OPT and Hamiltonian. This comprehensive approach broadens our understanding and empowers us to delve deeper into the complexities of point sets and their triangulations.

Dist. Matrix	v11	v12	v13	v14	v21	v22	v23
v11	3	11.14		6.10	2.83		
v12	1	4	11.66		8.54		4.47
v13		1	3	5.00			7.21
v14	1		1	5	4.47	2.83	6.08
v21	3	3		-3	5	2.00	6.40
v22				3	2	3	5.00
v23	3	3	3	3	2	2	5

Figure 2: The given matrix is indexed with vertices to document Dash Triangulation input. The diagonal contains the degrees of each vertex. The values above the diagonal indicate the distances and the values below the diagonal represent the sequence of edge insertion or deletion (the negative sign means the flipped edge).

The stochastic procedure, based on the LUDO game, enables the computation of a triangulation that meets multiple criteria or comprises parts developed according to several criteria. This can be achieved through a random approach involving edge insertion and flip operations. By randomly selecting edges and evaluating their insertion based on desired criteria, along with employing flip operations to improve the triangulation's quality, we have tried to achieve a mixture of criteria resulting in the optimal solution.

Formally, we can summarize, that TSC ⊄ DT and TSC ⊄ MWT hold for alternative triangulations as DT = MWT = GT = OPT = DashT (without the flip) and even for the reduced Dillencourt-6 pointset {A (4, 6), C (5, 0), D (11.1, 0.1), E (9, 2), F (7, 2), G (6, 10)}. TSC is contained in the flipped Dash edges, see Figures 1 and 2.

For the natural process, we employed the Lagrangian sheet membrane surface evolution [Čavarga, 2022], see Figure 3. This proof-of-the-concept is in the experimental stage up to now.

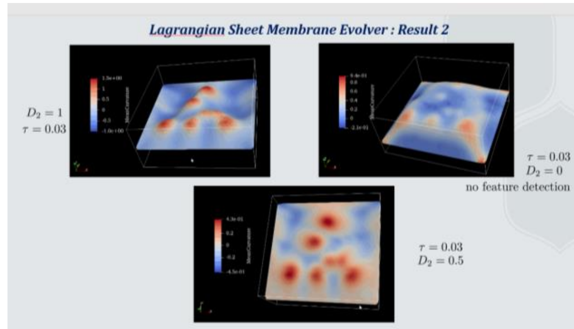

Figure 3: Lagrangian Sheet Membrane Evolver for Dillencourt-7 dataset elevation in 3D. A snapshot from the animation by Martin Cavarga presented at SVK April 19, 2023. Extremal and interior edges are indicated by local curvature and color visualization.

Additionally, we found promising results when utilizing the MWT heuristic with iterated Euclidean Minimum Matching. These findings suggest that this approach holds potential for future experiments.

3 Conclusion and Future Work

Our forthcoming research endeavours will focus on examining the potential inclusion of the TSP within multi-criteria triangulations. Specifically, we plan to investigate the applicability of stochastic multicriteria triangulations, as proposed in [Kolingerova, 2001], by evaluating their effectiveness using established TSP solutions. To accomplish this, we intend to utilize the well-known TSPLIB dataset, which provides a comprehensive collection of TSP instances with known solutions.

References

[Čavarga, 2022] ČAVARGA, M. 2022. Advection-Driven Shrink-Wrapping of Triangulated Surfaces. *Proceedings of the 26th Central European Seminar on Computer Graphics: CESCG 2022.* ISBN 978-3-9504701-3-0. Vienna: Institute of Visual Computing and Human-Centered Technology, 2022. pp. 95-104.[Online, March27,2023] https://cescg.org/cescg_submission/advection-driven-shrink-wrapping-of-triangulated-surfaces/

[Dillencourt, 1987] DILLENCOURT, M. B. 1987. Traveling Salesman Cycles are not Always Subgraphs of Delaunay Triangulations or of Minimum Weight Triangulations. *Inf. Process. Lett.* 1987, 24, 339–342.

[Kolingerova, 2001] KOLINGEROVA, I., FERKO, A. 2001. Multicriteria-optimized triangulations. *Visual Computer* 17 (2001) 6, 380-395.