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Shrink-Wrapping of Mesh Surfaces via Lagrangian Evolution

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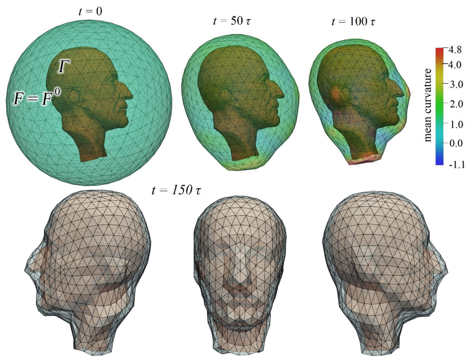

Figure 1: After computing signed distance field (SDF) of the *Bust of Max Planck* mesh Γ with voxel resolution $152 \times 176 \times 161$ an evolution of surface F (with initial condition F^0 as a geodesic icosahedron with subdivision level 3) is run with $N_t = 150$ time steps of length $\tau = 0.01$.

Methods based on diffusion often produce natural-looking shapes. Considering an *obstacle* for such process we simulate objects wrapped by a surface foil. We provide an outline of a shrink-wrapping algorithm of triangular meshes wrapping onto general polygonal meshes, with a sequence of pre-processing steps for computing the distance field.

Let F be a parametrization of an evolving surface in $\mathbb{R}^3$. Our model is based on a non-linear parabolic initial value problem

$$\partial_t F = \epsilon \Delta_{g_F} F + \eta N + v_T \ , \quad F(\cdot, 0) = F^0, \tag{1}$$

where Δ_{g_F} is the Laplace-Beltrami operator with respect to the current surface metric g_F, N is the outward-pointing unit normal to F, and ϵ, η are control functions for the two main components of evolution in the normal direction [2]:

$$\epsilon(d) := C_1\big(1 - \mathrm{e}^{-d^2/C_2}\big) \ , \quad C_1, C_2 > 0, \tag{2}$$

$$\eta(d) := Cd\big((-\nabla d \cdot N) - D\sqrt{1 - (\nabla d \cdot N)^2}\big), \quad C > 0, \ D \geq 0, \tag{3}$$

where d is the distance field of input surface $\Gamma \subset \mathbb{R}^3$. v_T represents tangential movement.

The discrete model of surface evolution uses a semi-implicit finite 2-volume scheme of (1), and is therefore subject to stability constraints. Our theoretical contribution is a heuristic based on time step size τ and finite 2-volume measures $\mu(V)$ of an evolving mesh with different levels of recursive 4-to-1 edge subdivision.

Under the simplified assumption with $\epsilon \equiv 1$, $\eta \equiv 0$ we need to scale the evolving surface by factor

$$\phi = \sqrt[3]{\tau / \mu_r(V)}, \tag{4}$$

where $\mu_r(V)$ is the mean measure of co volumes V of an expected *geodesic icosahedron* F^r with radius $r > 0$ at stopping time $t_s > 0$. Since individual areas of finite volume elements V covering F^r are approximately equal, we put

$$\mu_r(V) = 4\pi r^2 / N_V^s,$$

where

$$N_V^s = \big(N_E^0(4^s - 1) + 3N_V^0\big)/3. \tag{5}$$

Lagrangian evolution is first tested against ground truth with three numerical experiments for convergence rate, as in [3]. Likewise, model (1) is tested on the experimental data set of polygonal meshes for stability evaluation with scaling factor (4) and afterwards for final results [1] (see Fig. 2).

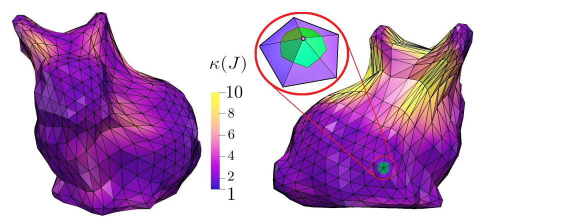

Figure 2: The condition number $\kappa(J)$ of the Jacobian J of map $\mathrm{Conv}(\{(0,0),(1,0),(0,1)\}) \mapsto \mathrm{Conv}(\{v_0^{u,v}, v_1^{u,v}, v_2^{u,v}\})$ (from unit triangle to the plane representation of each mesh triangle) evaluated at mesh vertices of a shrink-wrapped *Stanford bunny*. A 2-volume V around a mesh vertex is shown in green.

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References

[1] M. Čavarga, *Advection-driven shrink-wrapping of triangulated surfaces*, in CESCG Proceedings, url: `https://cescg.org/wp-content/uploads/2022/04/Cavarga-Advection-Driven-Shrink-Wrapping-of-Triangulated-Surfaces-3.pdf`, 2022.

[2] M. Huska, M. Medl'a, K. Mikula, and S. Morigi, *Lagrangian evolution approach to surface-patch quadrangulation*, Applications of Mathematics, 66 (2021), pp. 1–43.

[3] K. Mikula, M. Remešíkova, P. Sarkoci, and D. Ševčovič, *Manifold evolution with tangential redistribution of points*, SIAM Journal on Scientific Computing, 36 (2014), pp. A1384–A1414.